Big proof engines as little proof engines: new results on rewrite-based satisfiability procedures

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[1] Joint work with Alessandro Armando, Mnacho Echenim, Silvio Ranise, and Stephan Schulz

Decision procedures

- **Objective**: Decision procedures for application of automated reasoning to verification
- **Desiderata**: Efficient, scalable, expressive, proof-producing, easy to build, combine, extend, integrate, prove sound and complete
- **Issues**:
  - *Combination of theories:*
    usually done by combining procedures: complicated? *ad hoc*?
  - *Soundness and completeness proof:* usually *ad hoc*
  - *Implementation:* usually from scratch: correctness? integration in different environments? duplicated work?

"Little" engines and "big" engines of proof

- "Little" engines, e.g., validity checkers for specific theories
  Built-in theory, quantifier-free conjecture, decidable, combined by Nelson-Oppen scheme
- "Big" engines, e.g., general first-order theorem provers
  Any first-order theory, any conjecture, semi-decidable
- Not an issue of size (e.g., lines of code) of systems!
- Continuity: e.g., "big" engines may have theories built-in
- **Challenge**: can we get something good for decision procedures from big engines?

From a big-engine perspective

- *Combination of theories*: give union of presentations as input to prover
- *Soundness and completeness proof*: already given for first-order inference system
- *Implementation*: take first-order provers off the shelf
- *Proof generation*: already there by default
- *Model generation*: final T-sat set (starting point)
- **How to make it possible?**

Motivation

Rewrite-based satisfiability: new results

Rewrite-based methodology for T-satisfiability

Theories of equality, data structures, fragments integer arithmetic

General modularity theorem for combination of theories

Experimental appraisal

Comparison of E with CVC and CVC Lite

Synthetic benchmarks (valid and invalid): evaluate scalability

"Real-world" problems: huge sets of literals

Summary

What kind of theorem prover?

First-order logic with equality

$\mathcal{SP}$ inference system: rewrite-based

- *Simplification by equations*: normalize clauses
- *Superposition/Paramodulation*: generate clauses

Complete simplification ordering (CSO) $\succ$ on terms, literals and clauses: $\mathcal{SP}_\succ$

(Fair) $\mathcal{SP}_\succ$*-strategy* : $\mathcal{SP}_\succ$ + (fair) search plan

Rewrite-based methodology for T-satisfiability

- *T-satisfiability*: decide satisfiability of set S of *ground literals* in theory (or combination) T
- *Methodology*:
  - *T-reduction*: apply inferences (e.g., to remove certain literals or symbols) to get equisatisfiable *T-reduced* problem
  - *Flattening*: flatten all ground literals (by introducing new constants) to get equisatisfiable T-reduced *flat* problem
  - *Ordering selection and termination*: prove that any fair $\mathcal{SP}_{\succ}$-strategy terminates when applied to a $\mathcal{T}$-reduced flat problem, provided $\succ$ is *$\mathcal{T}$-good*
- Everything *fully automated* except for termination proof

Covered theories

- *Non-empty lists*, *arrays* with and without extensionality, *finite sets* with extensionality [Armando, Ranise, Rusinowitch 2003]
- *Records* with and without extensionality, *possibly empty lists*, *integer offsets*, *integer offsets modulo* [Armando, Bonacina, Ranise, Schulz 2005]
- *Equality* [Lankford 1975]

In experiments: arrays, records, integer offsets, integer offsets modulo, equality and combinations (queues, circular queues)

The theory of records

Sort $\mathrm{REC}(id_1 : \mathrm{T}_1, \ldots, id_n : \mathrm{T}_n)$

Presentation $\mathcal{R}$:

$$
\begin{array}{lcl}
\forall x, v. & \mathsf{rselect}_i(\mathsf{rstore}_i(x, v)) \simeq v & 1 \leq i \leq n \\
\forall x, v. & \mathsf{rselect}_j(\mathsf{rstore}_i(x, v)) \simeq \mathsf{rselect}_j(x) & 1 \leq i \neq j \leq n \\
\forall x, y. & (\bigwedge_{i=1}^{n} \mathsf{rselect}_i(x) \simeq \mathsf{rselect}_i(y) \supset x \simeq y) &
\end{array}
$$

where x and y have sort REC and v has sort T_i.
Extensionality is the third axiom.

Records: termination of $\mathcal{SP}$

$\mathcal{R}$-*reduction*: eliminate disequalities between records by resolution with extensionality + splitting.

$\mathcal{R}$-*good*: $t \succ c$ for all ground compound terms t and constants c.

Termination: case analysis of generated clauses (CSO plays key role).

Theorem: A fair $\mathcal{R}$-good $\mathcal{SP}_{\succ}$-strategy is a polynomial $\mathcal{R}$-satisfiability procedure (with or without extensionality).

The theory of integer offsets

Fragment of the theory of the integers:
s: successor   p: predecessor

Presentation $\mathcal{I}$:

$$\begin{array}{ll} \forall x. & \mathrm{s}(\mathrm{p}(x)) \simeq x \\ \forall x. & \mathrm{p}(\mathrm{s}(x)) \simeq x \\ \forall x. & \mathrm{s}^i(x) \not\simeq x \quad \text{for } i > 0 \end{array}$$

Infinitely many acyclicity axioms (Ac)
Remark: these axioms imply that s is *injective* (Inj)

$$\forall x, y.\ \mathrm{s}(x) \simeq s(y) \supset x \simeq y$$

Integer offsets: termination of $\mathcal{SP}$

$\mathcal{I}$-*reduction*: eliminate p by replacing $\mathsf{p}(c) \simeq d$ with $c \simeq \mathsf{s}(d)$: first two axioms no longer needed, provided *Inj* is added.
Bound the number of acyclicity axioms:
$\forall x.\, \mathsf{s}^i(x) \not\simeq x$ for $0 < i \leq n$
if there are n occurrences of s.

$\mathcal{I}$-*good*: $t \succ c$ for all constants c and terms t with top symbol s.

Termination: case analysis of generated clauses.

Theorem: A fair $\mathcal{I}$-good $\mathcal{SP}_{\succ}$-strategy is an exponential $\mathcal{I}$-satisfiability procedure (polynomial on *Ac* only).

The theory of integer offsets modulo

To reason with indices ranging over the integers mod k ($k > 0$)

Presentation $\mathcal{I}_k$:

$$\begin{array}{lll} \forall x. & \mathrm{s}(\mathrm{p}(x)) \simeq x & \\ \forall x. & \mathrm{p}(\mathrm{s}(x)) \simeq x & \\ \forall x. & \mathrm{s}^i(x) \not\simeq x & 1 \leq i \leq k-1 \\ \forall x. & \mathrm{s}^k(x) \simeq x & \end{array}$$

Finitely many axioms.

Integer offsets modulo: termination of $\mathcal{SP}$

$\mathcal{I}_k$*-reduction*: same as $\mathcal{I}$-reduction.

$\mathcal{I}_k$*-good*: same as $\mathcal{I}$-good.

Termination: case analysis of generated clauses.

Theorem: A fair $\mathcal{I}$-good $\mathcal{SP}_{\succ}$-strategy is an exponential $\mathcal{I}_k$-satisfiability procedure.

The theory of possibly empty lists

Presentation $\mathcal{L}$:

$$\begin{aligned}
\forall x, y.\ \ \mathsf{car}(\mathsf{cons}(x, y)) &\simeq x \\
\forall x, y.\ \ \mathsf{cdr}(\mathsf{cons}(x, y)) &\simeq y \\
\forall x, y.\ \ \mathsf{cons}(x, y) &\not\simeq \mathsf{nil} \\
\forall y.\ y \not\simeq \mathsf{nil} \supset \mathsf{cons}(\mathsf{car}(y), \mathsf{cdr}(y)) &\simeq y \\
\mathsf{car}(\mathsf{nil}) &\simeq \mathsf{nil} \\
\mathsf{cdr}(\mathsf{nil}) &\simeq \mathsf{nil}
\end{aligned}$$

Unsorted, possibly cyclic lists.

Possibly empty lists: termination of $\mathcal{SP}$

$\mathcal{L}$-reduction: none.

$\mathcal{L}$-good:

1. $t \succ c$ for all ground compound terms t and constants c,
2. $t \succ$ nil for all terms t with top symbol cons.

Termination: case analysis of generated clauses.

Theorem: A fair $\mathcal{L}$-good $\mathcal{SP}_{\succ}$-strategy is an exponential $\mathcal{L}$-satisfiability procedure.

A modularity theorem for combination of theories

- *Modularity*: if $\mathcal{SP}_{\succ}$-strategy decides $\mathcal{T}_i$-sat problems then it decides $\mathcal{T}$-sat problems for $\mathcal{T} = \bigcup_{i=1}^{n} \mathcal{T}_i$
- $\mathcal{T}_i$-reduction and flattening apply as for each theory
- Termination?

Three simple conditions

- $\succ$ $\mathcal{T}$-*good*, if $\mathcal{T}_i$-*good* for all i, $1 \leq i \leq n$
- The $\mathcal{T}_i$ do not share function symbols
  (*Intuition*: no paramodulation from compound terms across theories)
- Each $\mathcal{T}_i$ is *variable-inactive*:
  in no persistent clause $t \simeq x$ with $x \notin Var(t)$ is maximal
  (*Intuition*: no paramodulation from variables across theories, since for $t \simeq x$ where $x \in Var(t)$ it is $t \succ x$)

The modularity theorem

Theorem: if

- No shared function symbol (shared constants allowed),
- Variable-inactive theories $\mathcal{T}_i$, $1 \leq i \leq n$,
- A fair $\mathcal{T}_i$-good $\mathcal{SP}_{\succ}$-strategy is $\mathcal{T}_i$-satisfiability procedure,

then
a fair $\mathcal{T}$-good $\mathcal{SP}_{\succ}$-strategy is a $\mathcal{T}$-satisfiability procedure.

Equality, *arrays* (with or without extensionality), *records* (with or without extensionality), *integer offsets*, *integer offsets modulo* and *possibly empty lists* all satisfy these hypotheses.

A few remarks on generality I

- *Purely equational theories*:
  no trivial models $\Rightarrow$ variable-inactive
- *Horn theories*:
  no trivial models + maximal unit strategy $\Rightarrow$ variable-inactive
- *Maximal unit strategy*:
  restricts superposition to unit clauses and paramodulates unit clauses into maximal negative literals [Dershowitz 1990]

A few remarks on generality II

- *First-order theories*: variable-inactive excludes, e.g., $a_1 \simeq x \vee \ldots \vee a_n \simeq x$, a_i constants (*)
  Such a clause implies *not stably-infinite*, hence *not convex* under the no trivial models hypothesis:
  if $\mathcal{T}_i$ not variable-inactive for (*), Nelson-Oppen does not apply either.
- $\mathcal{T}$ *convex*:
  $\mathcal{T} \models H \supset \bigvee_{i=1}^{n} P_i$ implies $\mathcal{T} \models H \supset P_j$ for some j.
- $\mathcal{T}$ *stably infinite*:
  quantifier-free $\mathcal{T}$-formula has $\mathcal{T}$-model iff has infinite $\mathcal{T}$-model.

Experimental setting

- Three systems:
  - The E theorem prover: E 0.82 [Schulz 2002]
  - CVC 1.0a [Stump, Barrett and Dill 2002]
  - CVC Lite 1.1.0 [Barrett and Berezin 2004]
- Generator of pseudo-random instances of synthetic benchmarks
- 3.00GHz 512MB RAM Pentium 4 PC: max 150 sec and 256 MB per run
- **Folklore**: systems with built-in theories are out of reach for prover with presentation as input ...

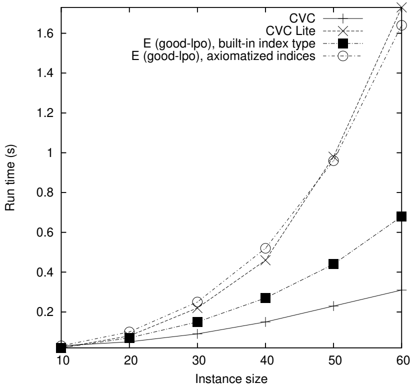
Synthetic benchmarks

- STORECOMM(n), SWAP(n), STOREINV(n): arrays with extensionality
- IOS(n): arrays and integer offsets
- QUEUE(n): records, arrays, integer offsets
- CIRCULAR_QUEUE(n, k): records, arrays, integer offsets mod k

STORECOMM(n), SWAP(n), STOREINV(n): both valid and invalid instances

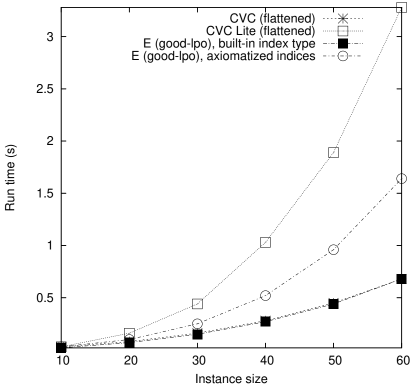
Parameter n: test *scalability*

Performances on valid STORECOMM(n) instances



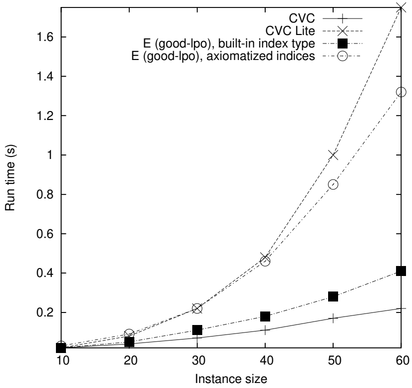
Native input: CVC wins but E better than CVC Lite

Performances on valid STORECOMM(n) instances



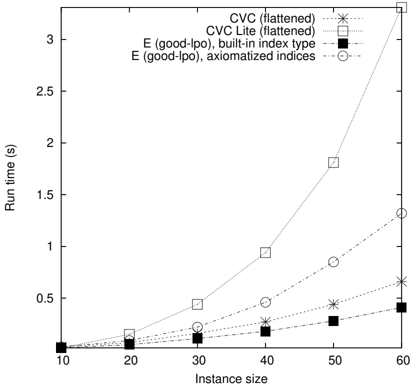
Flat input: E matches CVC

Performances on invalid STORECOMM(n) instances



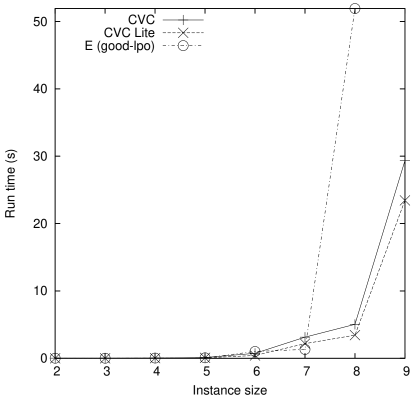
Native input: prover conceived for unsat handles sat even better

Performances on invalid STORECOMM(n) instances



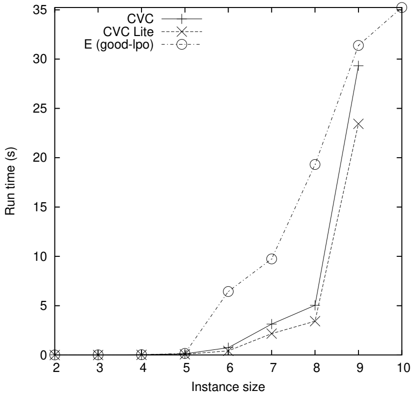
Flat input: E surpasses CVC

Performances on valid $\mathtt{SWAP}(n)$ instances



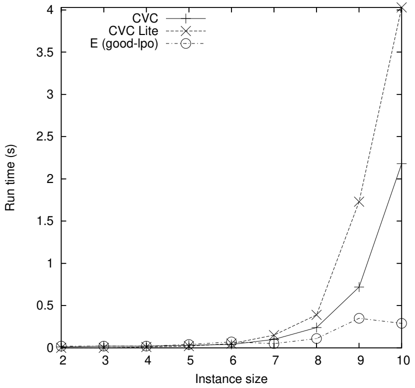
Harder problem: no system terminates for $n \geq 10$

Performances on valid SWAP(n) instances



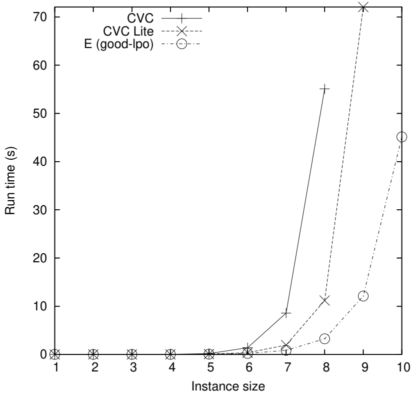
Added lemma for E: additional flexibility for the prover

Performances on invalid SWAP(n) instances



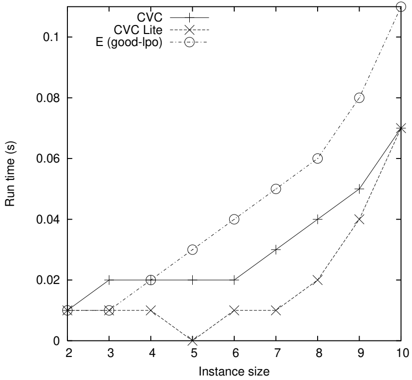
Easier problem, but E clearly ahead

Performances on valid STOREINV(n) instances



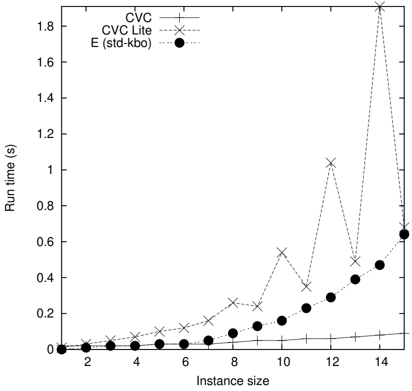
E(std-kbo) does it in *nearly constant time*!

Performances on invalid STOREINV(n) instances



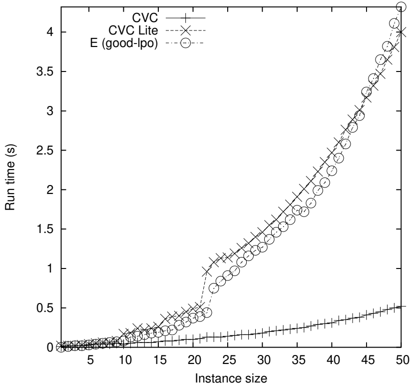
Not as good for E but run times are minimal

Performances on IOS instances



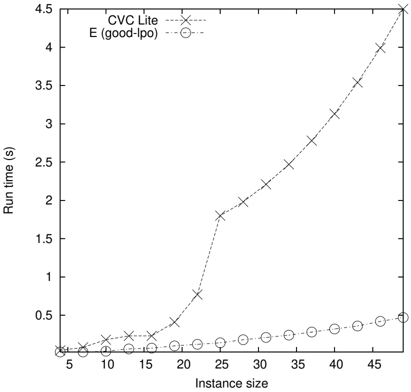
CVC and CVC Lite have built-in $\mathcal{LA}(\mathcal{R})$ and $\mathcal{LA}(\mathcal{I})$ respectively!

Performances on QUEUE instances (plain queues)



CVC wins (built-in arithmetic!) but E matches CVC Lite

Performances on $\texttt{CIRCULAR\_QUEUE}(n, k)$ instances $k = 3$



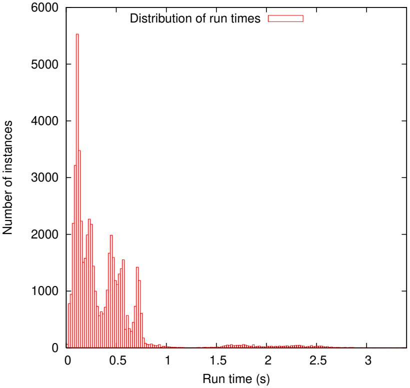
CVC does not handle integers mod k, E clearly wins

“Real-world” problems

- UCLID [Bryant, Lahiri, Seshia 2002]: suite of problems
- haRVey [Déharbe and Ranise 2003]: extract T-sat problems
- over 55,000 proof tasks: integer offsets and equality
- all valid

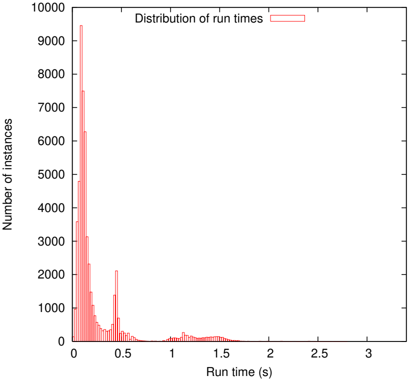
Test performance on huge sets of literals.

Run time distribution for *E(auto)* on UCLID set



Auto mode: prover chooses search plan by itself

Better run time distribution for E on UCLID set



Optimized strategy: found by testing on random sample of 500 problems (less than 1%)

Summary

- *General methodology* for rewrite-based T-sat procedures and its application to several theories relevant to verification
- *Modularity theorem* for combination of theories
- Experiments: first-order prover
  - *taken off the shelf* and
  - conceived for very different search problems

  compares amazingly well with built-in theories validity checkers

Current work

- Precise relationship between variable-inactive and stably-infinite, convex (e.g., $\mathcal{R}$ and $\mathcal{A}$, arrays, are not convex) [Bonacina, Ghilardi, Nicolini, Ranise, Zucchelli 2006]
- $\mathcal{T}$-satisfiability procedures for all theories of *recursive data structures*:
  one constructor and k selector ($k = 1$: integer offsets, $k = 2$: lists) [Bonacina, Echenim 2006]
- $\mathcal{T}$-decision procedures (arbitrary quantifier-free formulæ) [Bonacina, Echenim 2006]

Directions for future work

- Search plans for $\mathcal{T}$-sat problems
- Finer complexity results for specific search plans
- More or stronger termination results
- Integration with approaches for full $\mathcal{LA}$ or bit-vectors
- $\mathcal{T}$-decision procedures: integration with SAT-solver?
- Combination with automated model building
- In general: explore "big" engines technology for decision procedures

Big picture

Reasoning environments for verification (and more):

- SAT-solvers (e.g., DPLL, Stålmarck's method)
- "Little" engines
- "Big" engines (e.g., Rewrite-based, Stålmarck's method extended)
- Good interfaces
- ... ...